

ACT-04/97  
CTP-TAMU-05/97  
quant-ph/9702003

## Microtubules: The neuron system of the neurons?

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### Abstract

In this talk we review recent work on integrable models for Microtubule (MT) networks, subneural paracrystalline cytoskeletal structures, which seem to play a fundamental role in the neurons. We cast here the complicated MT dynamics in the form of a 1+1-dimensional non-critical string theory, which can be formulated in terms of (dual) Dirichlet branes, according to modern perspectives. We suggest that the MTs are the microsites in the brain, for the emergence of stable, macroscopic quantum coherent states, identifiable with the *preconscious states*. Quantum space-time effects, as described by non-critical string theory, trigger then an *organized collapse* of the coherent states down to a specific or *conscious state*. The whole process we estimate to take  $\mathcal{O}(1\text{ sec})$ , in excellent agreement with a plethora of experimental/observational findings. The complete integrability of the stringy model for MT proves sufficient in providing a satisfactory solution to memory coding and capacity. Such features might turn out to be important for a model of the brain as a quantum computer.

◇ Invited talk presented at the Workshop on “Biophysics of Microtubules”, Texas Medical Center, Houston, Texas, April 1996.

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The interior of living cells is structurally and dynamically organized by *cytoskeletons*, i.e. networks of protein polymers. Of these structures, *MicroTubules* (MT) appear to be [11] the most fundamental. These are paracrystalline cytoskeletal structures which seem to play a fundamental rôle for the cell *mitosis*, as well as for the transfer of electric signals and, more general, for dissipation-free energy transfer in the cell, according to ideas of Fröhlich (1986) [19]. MT networks are, therefore, also important for neuron cells. In this latter respect, according to recent ideas [20, 28], neuron MT are believed to be associated with conscious perception. This conjecture was based on the observation that certain micro-organisms without a nervous system but with cytoskeletal structure, c.f. *paramitium*, exhibit some sort of unexpected awareness. Some scenaria have been developed in order to provide physical mechanisms for such a phenomenon [28]. According to these, the MT networks may sustain *macroscopic* coherent quantum mechanical states, identified with the *preconscious states*. Coupling the latter to space-time quantum (fluctuating) gravitational degrees of freedom triggers an *organized collapse* down to a specific or *conscious* state. Such daring assumptions/conjectures have gain some support by the fact that in certain physical models of MT dynamics the estimated time of collapse is of order  $\mathcal{O}(1 \text{ sec})$ , which is in excellent qualitative agreement with a plethora of experimental/observational findings in Neurobiology.

In ref. [20], however, no specific physical model has been constructed, which could constitute a physical realization of the above-described phenomenon. Such an attempt was made in ref. [24], where there was presented a microscopic model for MT quantum dynamics, realizing the above scenario, yielding a collapse time of  $\mathcal{O}(1 \text{ sec})$ . It is the purpose of this talk to review the physical aspects of that work, and also to cast it in a more recent perspective following subsequent developments in the field of stringy quantum gravity, which occurred over the past year.

First, let us recapitulate the experimental findings concerning MT that will be useful in our analysis. MT are hollow cylinders comprised of an exterior surface (of cross-section diameter  $25 \text{ nm}$ ) with 13 arrays (protofilaments) of protein dimers called tubulines. The interior of the cylinder (of cross-section diameter  $14 \text{ nm}$ ) contains ordered water molecules, which implies the existence of an electric dipole moment and an electric field. The arrangement of the dimers is such that, if one ignores their size, they resemble triangular lattices on the MT surface. Each dimer consists of two hydrophobic protein pockets, and has an unpaired electron. There are two possible positions of the electron, called  $\alpha$  and  $\beta$  *conformations*. When the electron is in the  $\beta$ -conformation there is a  $29^\circ$  distortion of the electric dipole moment as compared to the  $\alpha$  conformation.

In standard models for the simulation of the MT dynamics, the ‘physical’ degree of freedom - relevant for the description of the energy transfer - is the projection of the electric dipole moment on the longitudinal symmetry axis (x-axis) of the

MT cylinder. The  $29^\circ$  distortion of the  $\beta$ -conformation leads to a displacement  $u_n$  along the  $x$ -axis, which is thus the relevant physical degree of freedom. This way, the effective system is one-dimensional (spatial), and one has a first indication that quantum integrability might appear. We shall argue later on that this is indeed the case.

Information processing occurs via interactions among the MT protofilament chains. The system may be considered as similar to a model of interacting Ising chains on a triangular lattice, the latter being defined on the plane stemming from fileting open and flatening the cylindrical surface of MT. Classically, the various dimers can occur in either  $\alpha$  or  $\beta$  conformations. Each dimer is influenced by the neighboring dimers resulting in the possibility of a transition. This is the basis for classical information processing, which constitutes the picture of a (classical) cellular automaton.

The quantum computer character of the MT network results from the assumption that each dimer finds itself in a superposition of  $\alpha$  and  $\beta$  conformations [20]. There is a macroscopic coherent state among the various chains, which lasts for  $\mathcal{O}(1 \text{ sec})$  and constitutes the ‘preconscious’ state [24]. The interaction of the chains with (stringy) quantum gravity, then, induces self-collapse of the wave function of the coherent MT network, resulting in quantum computation.

In ref. [24] we assumed, for simplicity, that the collapse occurs mainly due to the interaction of each chain with quantum gravity, the interaction from neighboring chains being taken into account by including mean-field interaction terms in the dynamics of the displacement field of each chain. This amounts to a modification of the effective potential by anharmonic oscillator terms. Thus, the effective system under study is two-dimensional, possessing one space and one time coordinate. The precise meaning of ‘time’ in our model will be clarified when we discuss the ‘non-critical string’ representation of our system.

Let  $u_n$  be the displacement field of the  $n$ -th dimer in a MT chain. The continuous approximation proves sufficient for the study of phenomena associated with energy transfer in biological cells, and this implies that one can make the replacement

$$u_n \rightarrow u(x, t) \tag{1}$$

with  $x$  a spatial coordinate along the longitudinal symmetry axis of the MT. There is a time variable  $t$  due to fluctuations of the displacements  $u(x)$  as a result of the dipole oscillations in the dimers. At this stage,  $t$  is viewed as a reversible variable. The effects of the neighboring dimers (including neighboring chains) can be phenomenologically accounted for by an effective double-well potential [32]

$$U(u) = -\frac{1}{2}Au^2(x, t) + \frac{1}{4}Bu^4(x, t) \tag{2}$$

with  $B > 0$ . The parameter  $A$  is temperature dependent. The model of ferroelectric distortive spin chains of ref. [7] can be used to give a temperature dependence

$$A = -|const|(T - T_c) \quad (3)$$

where  $T_c$  is a critical temperature of the system, and the constant is determined phenomenologically [32]. In realistic cases the temperature  $T$  is very close to  $T_c$ , which for the human brain is taken to be the room temperature  $T_c = 300K$ . Thus, below  $T_c$   $A > 0$ . The important relative minus sign in the potential (2), then, guarantees the necessary degeneracy, which is necessary for the existence of classical solitonic solutions. These constitute the basis for our coherent-state description of the preconscious state.

The effects of the surrounding water molecules can be summarized by a viscous force term that damps out the dimer oscillations,

$$F = -\gamma \partial_t u \quad (4)$$

with  $\gamma$  determined phenomenologically at this stage. This friction should be viewed as an environmental effect, which however does not lead to energy dissipation, as a result of the non-trivial solitonic structure of the ground-state and the non-zero constant force due to the electric field. This is a well known result, directly relevant to energy transfer in biological systems [23].

To determine a dynamical equation of motion for the MT dimers, each having a mass  $M$ , one should include a phenomenological kinetic term in the respective equation. It is also convenient [32, 24] to use a normalized displacement field

$$\psi(\xi) = \frac{u(\xi)}{\sqrt{A/B}} \quad (5)$$

where,

$$\xi \equiv \alpha(x - vt) \quad \alpha \equiv \sqrt{\frac{|A|}{M(v_0^2 - v^2)}} \quad (6)$$

with

$$v_0 \equiv \sqrt{k/M} R_0 \quad (7)$$

the sound velocity, of order  $1km/sec$ , and  $v$  the propagation velocity to be determined below. Above  $k$  is a stiffness parameter,  $R_0$  is the equilibrium spacing between adjacent dimers. In terms of the  $\psi(\xi)$  variable the equation of motion acquires the form of the equation of motion of an anharmonic oscillator in a frictional environment

$$\begin{aligned} \psi'' + \rho\psi' - \psi^3 + \psi + \sigma &= 0 \\ \rho &\equiv \gamma v [M|A|(v_0^2 - v^2)]^{-\frac{1}{2}}, \quad \sigma = q\sqrt{B}|A|^{-3/2}E \end{aligned} \quad (8)$$

where the prime denotes differentiation with respect to  $\xi$ , and  $E$  is the electric field due to the ‘giant dipole’ representation of the MT cylinder, as suggested by the experimental results [32], and  $q = 18 \times 2e$  ( $e$  the electron charge) is a mobile charge.

Equation (8) has a *unique* bounded solution [32]

$$\psi(\xi) = a + \frac{b - a}{1 + e^{\frac{b-a}{\sqrt{2}}\xi}} \quad (9)$$

with the parameters  $b, a$  and  $d$  satisfying:

$$(\psi - a)(\psi - b)(\psi - d) = \psi^3 - \psi - \left( \frac{q\sqrt{B}}{|A|^{3/2}} E \right) \quad (10)$$

According to ref. [23] the importance of the force term  $qE$  lies in the fact that eq (8) admits displaced classical soliton solutions (kinks) with no energy loss. The kink propagates along the protofilament axis with fixed velocity

$$v = v_0 \left[ 1 + \frac{2\gamma^2}{9d^2 M v_0^2} \right]^{-\frac{1}{2}} \quad (11)$$

This velocity depends on the strength of the electric field  $E$  through the dependence of  $d$  on  $E$  via (10). Notice that, due to friction,  $v \neq v_0$ , and this is essential for a non-trivial second derivative term in (8), necessary for wave propagation. For realistic biological systems  $v \simeq 2m/sec$ . With a velocity of this order, the travelling waves of kink-like excitations of the displacement field  $u(\xi)$  transfer energy along a moderately long microtubule of length  $L = 10^{-6}m$  in about

$$t_T = 5 \times 10^{-7} sec \quad (12)$$

This time is very close to Frohlich’s time for coherent phonons in biological system [19]. We shall come back to this issue later on.

The total energy of the solution (9) is easily calculated to be [32]

$$E = \frac{1}{R_0} \int_{-\infty}^{+\infty} dx H = \frac{2\sqrt{2}}{3} \frac{A^2}{B} + \frac{\sqrt{2}}{3} k \frac{A}{B} + \frac{1}{2} M^* v^2 \equiv \Delta + \frac{1}{2} M^* v^2 \quad (13)$$

which is *conserved* in time. The ‘effective’ mass  $M^*$  of the kink is given by

$$M^* = \frac{4}{3\sqrt{2}} \frac{MA\alpha}{R_0 B} \quad (14)$$

The first term of equation (13) expresses the binding energy of the kink and the second the resonant transfer energy. In realistic biological models the sum of these two terms dominate over the third term, being of order of  $1eV$  [32]. On the other hand, the effective mass in (14) is [32] of order  $5 \times 10^{-27} kg$ , which is about the proton mass ( $1GeV$ ) (!). As we discussed in ref. [24], and shall discuss later on, these values are essential in yielding the correct estimates for the time of collapse of the ‘preconscious’ state due to our quantum gravity environmental entangling.

To make plausible a consistent study of such effects, one has to represent the equation of motion (8) as being derived from string theory. In such a framework, and in particular in a non-critical string context [2, 8], the authors together with John Ellis [13] had already developed a theory of information storage in quantum space-times with (black-hole) singularities. The theory implies an irreversible time evolution for the matter system in interaction with the gravitational environment in a quite natural and mathematically rigorous way. The purpose of ref. [24] was to apply these ideas to the dynamics of brain MT in an attempt to present a semi-microscopic model for quantum brain function in the spirit of refs. [20, 28].

For this purpose, it is important to notice that the relative sign (+) between the second derivative and the linear term in  $\psi$  in equation (8) is such that this equation can be considered as corresponding to the tachyon  $\beta$ -function equation of a (1 + 1)-dimensional string theory, in a flat space-time with a dilaton field  $\Phi$  linear in the *space-like* coordinate  $\xi$  [2],

$$\Phi = -\rho\xi \quad (15)$$

Indeed, the most general form of a ‘tachyon’ deformation in such a string theory, compatible with conformal invariance is that of a travelling wave [25]  $T(x')$ , with

$$\begin{aligned} x' &= \gamma_{v_s}(x - v_s t) & ; & & t' &= \gamma_{v_s}(t - v_s x) \\ \gamma_{v_s} &\equiv (1 - v_s^2)^{-1/2} \end{aligned} \quad (16)$$

where  $v_s$  is the propagation velocity of the string ‘tachyon’ background. As argued in ref. [25] these translational invariant configurations are the most general backgrounds, consistent with a *unique* factorisation of the string  $\sigma$ -model theory on a Minkowski space-time  $G_{\mu\nu} = \eta_{\mu\nu}$

$$S = \frac{1}{4\pi\alpha'} \int d^2z \left[ \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu}(X) + \Phi(X) R^{(2)} + T(X) \right] \quad (17)$$

into two conformal field theories, for the  $t'$  and  $x'$  fields, corresponding to central charges

$$c_{t'} = 1 - 24v_s^2\gamma_{v_s}^2 \quad ; \quad c_{x'} = 1 + 24\gamma_{v_s}^2 \quad (18)$$

In our case (8), the rôle of the space-like coordinate  $x'$  is played by  $\xi$  (6). The velocity of light in this effective string model is replaced by the sound velocity  $v_0$  (7), and the velocity  $v_s$  is defined in terms of the velocity  $v$  of the kink (11) by expressing [25] the friction coefficient  $\rho$  in terms of the central charge deficit (18)

$$\rho = \sqrt{\frac{1}{6}(c(\xi) - 1)} = 2\gamma_{v_s} \quad (19)$$

The space-like ‘boosted’ coordinate  $\xi$ , thus, plays the rôle of space in this effective/Liouville mode string theory.

Notice that with the definition (19) the local-field-theory kink solution (9), propagating with a *real* velocity  $v$ , is mapped to a non-critical string background which propagates with a velocity  $v_s$  that could be *imaginary* ( $v_s^2 < 0$ ). Indeed, the condition for reality of  $v_s$  can be easily found from (8,11,19) to be

$$8M|A| < 9d^2Mv_0^2 \quad (20)$$

It can be easily seen that for the generic values of the parameters of the MT model described above [32], (20) is not satisfied, which implies that, when formulated as a non-critical string theory, the MT system corresponds to a 1 + 1 dimensional non-critical string with Wick-rotated ‘time’  $s = it$  [25], and, therefore, corresponds to a matter central charge that overcomes the  $c = 1$  barrier

$$25 \geq c_{s'} = 1 + 24|v_s|^2(1 + |v_s|^2)^{-1} \geq 1 \quad (21)$$

In this regime, Liouville theory is poorly understood, but there is the belief that this range of matter central charges is characterized by polymerization properties of random surfaces. From our point of view this may be related to certain growth properties of the MT networks, which are discussed in ref. [24].

An interesting question arises as to whether there are circumstances under which (20) is satisfied, in which case the matter content of the theory is characterized by the central charge  $c_{\nu'} \leq 1$  (18). To this end, we note that the parameters with the largest uncertainty in the relation (20) are  $A$  and  $d$ . At present, there are no accurate experimental data for  $A$ , which depends on temperature (3). The parameter  $d$  is sensitive to the order of magnitude of the electric field  $E$ . The latter is non-uniform along the MT axis. There is a sharp increase of  $E$  towards the end points of the MT protofilament axis [32], and for such large  $E$ ,  $d \sim E^{1/3}$ . Due to (11), an increase in  $d$  results in an increase in the kink velocity  $v$  at the end points of the MT. For realistic biological systems, under normal circumstances, the increase in  $v$  can be even up to two orders of magnitude, resulting in kink velocities of  $\mathcal{O}[10^2 m/sec]$  [32] at the end points of the MT.

As we shall argue now, both parameters  $A$  and  $d$  can be drastically affected by an abrupt distortion of the environment due to the influence of an *external stimulus*. For instance, it may be possible for such abrupt distortions to cause a local disturbance among the dimers, so that the value of  $A$  is momentarily diminished significantly and the condition (20) is met. However, such a distortion might affect the order of magnitude estimates of the effective mass scales involved in the problem, as we discussed above (14), and this may have consequences for the decoherence time. So, we shall not consider it for the purposes of this work. More plausibly, an abrupt environmental distortion will lead to a sudden increase in the electric field  $E$ , which, in turn, results in the formation of a fast kink of  $v \sim v_0$ , via an increase in  $d$ . As we have seen above, this is not unreasonable for the range of the parameters pertaining to realistic biological systems. In such cases the (fast) kinks are represented as

translational-invariant backgrounds of non-critical string theories, propagating with *real* velocities  $v_s$  (19). This is the kind of structures that we shall mostly be interested in for the purposes of this work. Coupling them to quantum gravity will lead to the collapse of the preconscious state, as we shall discuss in the next section.

The important advantage of formulating the MT system as a  $c = 1$  string theory, lies in the possibility of casting the friction problem in a Hamiltonian form [13, 24]. This is quite important for the *canonical* quantization of the kink solution, which will provide us with a concrete example of a large-scale quantum coherent state for the preconscious state [24]. An explicit construction of such quantum solitonic coherent states has been given in ref. [24]. Such large-scale coherent states in the MT networks may, thus, be considered responsible for loss-free energy transfer along the tubulines.

Suppose now that *external stimuli* produce sufficient distortion in the electric dipole moments of the water environment of the MT. As a result, conformational (quantum) transitions of the tubulin dimers occur. Such abrupt pulses may cause sufficient distortion of the space time surrounding the tubulin dimers, which in turn leads to the formation of *virtual* ‘black holes’ in the effective target two-dimensional space time. Formally this is expressed by coupling the  $c = 1$  string theory to two-dimensional quantum gravity [24]. This elevates the matter-gravity system to a critical  $c = 26$  theory. Such a coupling, then, causes decoherence, due to induced instabilities of the kink quantum-coherent ‘preconscious state’, in a way described in ref. [24]. As the required collapse time of  $\mathcal{O}(1\text{sec})$  of the wave function of the coherent MT network is several orders of magnitude bigger than the energy transfer time  $t_T$  (12), the two mechanisms are compatible with each other. Energy is transferred during the quantum-coherent preconscious state, in  $10^{-7}\text{sec}$ , and then collapse occurs to a certain (classical) conformational configuration. In this way, Frohlich’s frequency [19] associated with coherent ‘phonons’ in biological cells is recovered, but in a rather different setting.

Once a virtual black hole is formed in a MT chain, the subsystem of the displacement modes  $\psi(\xi)$  becomes *open* in a statistical mechanics sense. This subsystem is a (1+1)-dimensional non-critical (Liouville) theory. It is known that the singularity structure of black holes in such systems is described by a topological  $\sigma$ -model, obtained by twisting the  $N = 2$  supersymmetric black hole [14]. The field theory *at the singularity* is described by an enhanced topological  $W_{1+\infty} \otimes W_{1+\infty}$  symmetry. *Away* from the singularity, such symmetry is *broken spontaneously* down to a single  $W_{1+\infty}$  symmetry, as a result of the non-vanishing target-space gravitational condensate [14]. Spontaneous breaking in a (1+1)-dimensional target string theory is allowed, in the sense that the usual infrared infinities that prevented it from happening in a local field theory setting are absent. In ref. [14, 13] we have demonstrated this phenomenon explicitly by showing that, due to the (twisted)  $N = 2$  supersymmetry associated with the topological nature of the singularity, there is a suppression



of the tunneling effects, which in point-like theories would prevent the phenomenon from occurring. As a result, the appearance of *massless* states takes place. Such states are delocalized global states, belonging to the lowest-level of the string spectrum [14]. These are the leg-poles that appear in the scattering amplitudes of  $c = 1$  Liouville theory [30]. Their excitation in the brain results in conscious perception, in a way similar to the one argued in the context of local field theory regarding the excitation of the dipole wave quanta [9]. Here, however, the mechanism of conscious perception appears formally much more complicated, due to the complicated nature of the enormous stringy symmetries that are spontaneously broken in this case.

From a formal point of view, the formation of virtual black holes, with varying mass, can be modelled by the action of world-sheet instanton deformations [13]. The latter have the property of shifting (renormalizing) the Wess-Zumino level parameter  $k$  (related to central charge) of the black hole conformal field theory of ref. [36]. From a conformal field theory point of view, instantons are associated with induced *extra logarithmic* divergencies (on the world sheet) in the presence of the matter leg-poles. In our approach to target time,  $t$ , as a dynamical world-sheet renormalization group scale [13],  $\phi = -t$ , such logarithmic divergencies, when regularized, lead to extra time dependences in the central charge of the theory, and hence to a time-dependent ‘effective’  $k$ , as mentioned above. It can be shown [13] that in such a case the *ADM* mass of the black hole [26] depends on the scale (time)

$$M_{bh} \propto \frac{1}{\sqrt{k(t) - 2}} e^a \quad (22)$$

where  $a$  is a constant that can be added to the dilaton field without affecting the conformal invariance of the black hole solution without matter. As shown in [13],  $k(t)$  actually increases with time  $t$ , leading asymptotically to  $\infty$ , which corresponds to the flat space-time limit. In that case, the system keeps ‘memory’ of the dilaton constant  $a$ , which pre-existed the black hole formation. From a MT point of view, the constant  $a$  corresponds to a spontaneously chosen vacuum string state as a result of spontaneous breaking mechanisms of, say, electric dipole rotational symmetries etc, in the ordered water environment [9]. This is important for *coding* of memory states in this framework. From the above discussion it becomes clear that *memory* operation in our approach is a two step process: (i) formation of a black hole, of a fixed value for the dilaton vacuum expectation value (vev)  $a$ , related to spontaneous breaking occurring in the water environment, as a result of an *external stimulus*, and (ii) evaporation of the black hole, due to *quantum* instabilities, described formally by world sheet instanton effects in our completely integrable approach to MT dynamics. The latter effects drive the black hole to a vanishing-mass limit by shifting  $k$  and *not* the dilaton vev,  $a$ . This implies *storage* of information, according to the general ideas of ref. [13]. Indeed, from a conformal field theory point of view, the constant  $a$  can be shifted by exactly marginal deformations (moduli) of the black hole  $\sigma$  model, whose couplings are *arbitrary*. Such an operator has been constructed explicitly in

ref. [6], and consists exclusively of combinations of global state deformations. It constitutes one of the (infinite number of)  $W_\infty$  charges that have been conjectured to characterize a two-dimensional stringy black hole [13].

We now consider the propagation of low-energy matter modes,  $T(\xi)$ , in such black hole space times. From the point of view of  $(1+1)$ -dimensional string theory such modes correspond to lowest-lying string states, known as ‘tachyons’. In our stringy representation of MT such excitations correspond to the displacement field  $\psi(\xi)$  [24]. The important point to notice is that the system of  $T(\xi)$  coupled to a black hole space-time, even if the latter is a virtual configuration, cannot be critical (conformal invariant) *non-perturbatively* if the tachyon has a travelling wave form. The factorisation property of the world-sheet action (17) in the flat target space-time case breaks down due to the non-trivial target-space graviton structure. Then a travelling wave cannot be compatible with conformal invariance, and renormalization scale dressing appears necessary. The gravity-matter system is viewed as a  $c = 26$  string [36, 13], and hence the renormalization scale is *time-like* [2]. This implies time dependence in  $T(\xi, t)$ .

A natural question arises whether there exist a deformation that turns on the coupling  $T(\xi)$  which is exactly marginal so as to maintain conformal invariance. The exactly marginal deformation of this black hole background that turns on matter,  $L_0^1 \bar{L}_0^1$  in the notation of ref. [6], couples necessarily the propagating tachyon  $T(\xi)$  zero modes to an infinity of higher-level string states [6]. The latter are classified according to discrete representations of the  $SL(2, R)$  isospin, and together with the propagating modes, form a target-space  $W_\infty$ -algebra [13, 3]. This coupling of massive and massless modes is due to the non-vanishing Operator Product Expansion (O.P.E.) among the vertex operators of the  $SL(2, R)/U(1)$  theory [6]. This theory possesses an infinity of conserved charges in target space [13] corresponding to the Cartan subalgebra of the infinite-dimensional  $W_\infty$  [3]. In practice, such global charges, which contribute phase factors to the string universe wave function, are impossible to measure by localized scattering experiments in our world. This, as explained in [13], leads to the effective breakdown of quantum coherence in the low-energy world [13]. From the point of view of MT, such  $W$  modes might be thought of as constituting the ‘*consciousness degrees of freedom*’ [24], which in this picture, are not exotic, as suggested in ref. [27], but exist already in a string formalism, and they result in the *complete integrability* of the two-dimensional black hole Wess-Zumino model.

This integrability persists quantization [37], and it is very important for the quantum coherence of the string black hole space-time [13]. Due to the specific nature of the  $W_\infty$  symmetries, there is no information loss during a stringy black hole decay, the latter being associated with instabilities induced by higher-genus effects on the world-sheet [15, 13]. The phase-space volume of the effective field theory is

preserved in time, *only if* the infinite set of the global string modes is taken into account. This is due to the string-level mixing property of the  $W_\infty$  - symmetries of the target space.

However, any local operation of measurement, based on local scattering of propagating matter, such as the functions performed by the human brain, will necessarily break this coherence, due to the truncation of the string deformation spectrum to the localized propagating modes  $T(\xi)$ . The latter will, then, constitute a subsystem in interaction with an *environment* of global string modes. The quantum integrability of the full string system is crucial in providing the necessary couplings. This breaking of coherence results in an arrow of time/Liouville scale, in the way explained briefly above [13]. The black-hole  $\sigma$ -model is viewed as a  $c = 26$  critical string, while the travelling wave background is a non-conformal deformation. To restore criticality one has to dress  $T(\xi)$  with a Liouville time dependence  $T(\xi, t)$  [13]. We stress, once more, that the Liouville renormalization scale now is time-like, in contrast with the previous string picture of a  $c = 1$  matter string theory, representing the displacement field  $\psi$  alone before coupling to gravity. In this framework, one can derive [13] an equation for the temporal evolution of the density matrix  $\rho$  of the subsystem comprising of the Liouville-dressed displacement field  $T(\xi, t)$  in interaction with the fluctuating space-time. The equation is of the generic form expected in an effective theory of quantum gravity [12]:

$$\partial_t \rho = i[\rho, H] + \delta H \rho \quad (23)$$

where  $\delta H$  is a non-commutator term, depending on data of the (non-critical)  $\sigma$ -model that describes propagation of the string excitations in a non-trivial black-hole space time [13, 24].

One can calculate in this approach the off-diagonal elements of the density matrix in the string theory space  $u^i$ , with now  $u^i(t)$  representing the displacement field of the  $i$ -th dimer [24]. The computation proceeds analogously [13] to the Feynman-Vernon [18] and Caldeira and Leggett [4] model of environmental oscillators, using the influence functional method, generalized properly to the string theory space  $u_i$ . The general theory of time as a world-sheet scale predicts [13] the following order-of-magnitude estimate for the decoherence time:

$$t_{col} = O\left[\frac{M_{gus}}{E^2 N}\right] \quad (24)$$

where  $E$  is a typical energy scale in the problem, and  $M_{gus}$  is a string scale, characterising the dynamics of the space-time environment of the effective MT model. If we take  $M_{gus}$  to be the realistic four-dimensional quantum string gravity scale  $10^{18}$  GeV [24], then we can estimate that a collapse time of  $\mathcal{O}(1 \text{ sec})$  is compatible with a number of coherent tubulins of order

$$N \simeq 10^{12} \quad (25)$$

provided that the energy stored in the kink background is of the order of  $eV$ . This is indeed the case of the (dominant) sum of binding and resonant transfer energies  $\Delta \simeq 1eV$  (13) at room temperature in the phenomenological model of ref. [32]. This number of tubulin dimers corresponds to a fraction of  $10^{-7}$  of the total brain, which is pretty close to the fraction believed to be responsible for human perception on the basis of completely independent biological methods.

Having described the basics of our mechanism for conscious perception, we went on in ref. [24] to discuss issues related to memory *coding* and *capacity* of the brain as a quantum computer. For this purpose one should first stress the importance of the surrounding water molecules for the proper functioning or even the existence of MT [9]. As a result of its electric dipole structure, the ordered water environment has been conjectured to exhibit a laser-like behaviour in ref. [10]. In local field theory models of the dynamics of MT [31, 33, 9] coherent modes emerge as a result of the interaction of the electric dipole moments of the water molecules with the quantized electromagnetic radiation. Such quanta can be understood [9] as Goldstone modes arising from the spontaneous breaking of the electric dipole symmetry, which in the work of ref. [9] was the only symmetry to be assumed spontaneously broken. In our string model, as discuss in [24], and mentioned above a more complicated (infinite-dimensional)  $W_\infty \otimes W_\infty$  symmetry breaks spontaneously, which incorporates the simple rotational symmetry of the point-like theory models. The emergence of coherent dipole quanta resembles the picture of Fröhlich coherent ‘phonons’ [19], emerging in biological systems for energy transfer without dissipation.

In our non-critical-string approach the existence of such coherent states in the surrounding water results in the friction term proportional to  $\rho$  in (8). What we have argued above is that, because of this interaction, a kink soliton can be formed, provided that the MT are of sufficient length. Such solitons can then be themselves squeezed coherent states, being responsible for a *preconscious* state of the mind. In local field theory models of the brain *external stimuli* are believed responsible for triggering spontaneous breaking of the electric dipole rotational symmetry of the water environment. The collective Goldstone modes of such a breaking (dipole quanta) are spin wave quanta and the system’s phases are macroscopically characterized by the value of the order parameter, which in this case is the polarization (*coding*). If the ground state of the system is considered as the *memory* state, then the above process is just *memory printing*. In this picture, *memory recall* corresponds to the excitation, under another external stimulus, of dipole quanta of similar nature to those leading to the printing. The brain, then, ‘consciously feels’ [33, 35] the pre-existing order in the ground state.

The main problem encountered in most of the local field theory models of the brain is related to memory capacity, as explained in [24], since those models contain only a *finite* amount of conserved quantum numbers, and therefore are insufficient

to store the enormous amount of information believed to be stored by the human brain. An attempt to overcome those difficulties has been made in the interesting work of ref. [35], making use of *dissipative* models for brain function, within a local field theory framework. As observed in ref. [35], the doubling of degrees of freedom which appears necessary for a canonical quantization of an open system in a dissipative environment [31, 34], is essential in yielding [5] a *non-compact*  $SU(1, 1)$  symmetry for the system of damped harmonic oscillators, used as a toy example for simulating quantum brain physics. The quantum numbers of such a system are the  $SU(1, 1)$  isospin and its third component,  $j \in Z_{\frac{1}{2}}, m \geq |j|$ . The memory (ground) state corresponds to  $j = 0$  and there is a huge degeneracy characterised by the various coexisting (infinite) eigenstates of the Casimir operator for the  $SU(1, 1)$  isospin. The open-character of the system introduces a time arrow which is associated with the *memory printing* process and is compatible with the ‘observation’ that ‘only the past can be recalled’ [35]. As far as we can see, the problem with this approach is that it necessarily introduces dissipation in the energy functional, through the non-hermitian terms in the interaction hamiltonian between the subsystem and the environment [31, 35]. Hence, it is not easy to see how to reconcile this with the above-mentioned property of biological systems to transfer energy without dissipation across the cells [19, 23]. Moreover, from our point of view, this approach cannot take into account realistic quantum gravity effects, which according to the hypothesis of refs. [20, 28] are considered responsible for conscious perception.

Our Stringy approach to the MT dynamics [24] seems to provide a way out of these problems [13] due to the infinite-dimensional gauge stringy symmetries that mix the various levels. In the (completely integrable) black hole model of ref. [36], which is used to simulate the physics of the MT [32], there is an underlying world-sheet  $SL(2, R)$  symmetry of the  $\sigma$ -model, according to which the various stringy states are classified. The various states of the model, including global string modes characterised by discrete values of (target) energy and momenta, are classified by the non-compact isospin  $j$  and its third component  $m$ , which - unlike the compact isospin  $SU(2)$  case - is not restricted by the value of  $j$ . Thus, for a given  $j$ , which in the case of string states plays the role of energy, one can have an *infinity* of states labelled by the value of the third isospin component  $m$ . All such states are characterised by a  $W_{1+\infty}$  symmetry in target space. As we mentioned above, this symmetry is responsible for the maintenance of *quantum coherence* in the presence of a black hole background [13], in the sense of an area-preserving diffeomorphism in a matter phase-space of the two-dimensional target space theory. Thus, for each matter ground state of a propagating degree of freedom, say the zero mode of the massless field corresponding to the static “tachyon” background of ref. [36], with  $SL(2, R)$  quantum numbers  $j = -\frac{1}{2}, m = 0$ , there will be an infinite (energy) degeneracy corresponding to a continuum of black hole space-time backgrounds with different *ADM* masses. These backgrounds are essentially generated by adding various constants to the configuration of the dilaton field in this two-dimensional string theory [36, 6]. It should be noted here that the infinity of propagating “tachyon”

states (lowest string mass-level (massless) states), corresponding to other values of  $m$ , for continuous representations of  $j$ , constitute *excitations* about the ground state(s), and, thus, they should not be considered as contributing to the ground state degeneracy. In principle, there may be an additional infinity of quantum numbers corresponding to higher-level  $W$ -hair charges of the black hole space time which are believed [13] responsible for quantum coherence at the full string theory level.

Taking into account the conjecture of the present work, that formation of virtual black holes can occur in brain MT models, which would correspond to different modes of collapse of pulses of the displacement field  $\psi$  defined in (5), one obtains a system of *coding* that is capable to solve in principle the problem of memory capacity. Information is stored in the brain in the following sense: every time there is an external stimulus that brings the brain out of equilibrium, one can imagine an abrupt conformational change of the MT dimers, leading to a collapse of the pulse pertaining to the displacement field. Then a (virtual) black hole is formed leading to a spontaneous collapse of the MT network to a ground state characterised by say a special configuration of the displacement field  $(j, m)$ . This ground state will be conformally invariant, and therefore a true vacuum of the string, only after complete evaporation of the black hole, which however would keep memory of the particular collapse mode in the ‘value’ of the constant added to the dilaton field, or other  $W$ -charges. This reflects the existence of additional exactly marginal deformations, consisting of global modes only, that are not directly accessible by local scattering experiments, in the context of the low energy theory of propagating modes (displacement field configurations  $\psi$ ). In such a case, the resulting ground state will be infinitely degenerate, which would solve the problem of *memory capacity*

Breaking of this degeneracy, can be achieved by means of an *external stimulus*, which is believed to be due to a weak field [9]. For instance, following the suggestion of ref. [9], we may imagine that an external weak field produces a spontaneous breaking of the electric dipole rotational symmetry in the water molecules, resulting in a ‘lasering’ [10] of the environmental surroundings of the MT system (excitation of coherent dipole quanta). Such an excitation of collective modes results in a specific *code* characterising the ground state, as we mentioned above. The so selected ground state of the ordered water molecules affects the MT chains, due to the friction coupling  $\rho$  in (8), (15). As becomes clear from the analysis above ((8)-(11)), the effects of the environment are described by selecting a specific value for the vacuum expectation value (condensate) of the dilaton field in our suggested stringy approach to MT dynamics. The other  $W$ -charges (moduli) may also be selected this way, which we believe corresponds to the *memory printing* process, i.e. storage of information by a selection of a given ground state. A new information would then choose a different value of the dilaton field or other  $W$ -hair charges, etc. This provides a new and satisfactory mechanism of *memory recall* in the following sense: if a new pulse happens to correspond to the same set of (conserved)  $W$ -hair moduli configurations [13], then the associated virtual black hole will be characterised by the

same set of quantum hair, and then the same memory state is reached *asymptotically* (process of ‘memory recall’). The discussion we gave in the previous section about the rôle of world-sheet instanton deformations in shifting the *ADM* mass of a black hole (22), while keeping memory of the dilaton v.e.v., finds a natural application in this coding process. Moreover, the irreversible arrow of time, endemic in Liouville string theory [13] explains naturally why “only the past can be recalled” [35].

To understand why the above process leads to a special coding, and how time reversal is spontaneously broken, as a result of this coding, it is sufficient to recall our discussion above, according to which in the presence of a space-time foamy environment, characterised by the virtual appearance and evaporation of black holes, there is a coupling of global modes to the propagating modes. As a result of the exactly marginal character of the deformations [13, 6], which thus respect conformal invariance at a string level, the environmental global modes match in a special way with the propagating mode  $j = -\frac{1}{2} m = 0$ , which is the zero mode of the (massless) tachyon corresponding to the tachyon background of a two dimensional black hole which constitutes the *ground* state or *memory* state of our system. This is a *special coding* which were it not for the infinite degeneracy of the black hole space time would lead to a restricted memory capacity<sup>1</sup>.

We cannot resist in pointing out that the existence of such coded situations in brain cells bears an interesting resemblance with DNA coding, with the important difference, however, that here it occurs in the model’s state space. In this context, we note that the genetic diversity is not due to an infinite number of nucleotide types, since in nature the relevant ones are only four, paired by two ( $A = T/G \equiv C$ ), but rather to a macroscopically large number of existing combinations in the DNA helix. Similarly, for the extremely rich (macroscopic) memory capacity in our stringy MT model, we may not need the full (infinite) set of the *W*-hair charges, but just the dilaton vev  $\langle \Phi \rangle$  may be sufficient as a ‘collective’ mode. The latter is, as we mentioned earlier, related to external stimuli through the equations (8)-(11).

A final comment we would like to make, concerns quantum uncertainties in the length of MT chains. In ref. [24] a prediction has been made, based on the stochasticity of the Liouville string theory [13], about quantum jumps of the tips of MT chains, as a result of quantum uncertainties. In a subsequent article [1] we have studied the propagation of light signals in a Liouville gravity environment, and argued that there are induced bounds in measuring distances,  $L$ , as a result of quantum-mechanical uncertainties, which behave like

$$\min\{\delta L\} \sim \sqrt{LL_s} \quad (26)$$

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<sup>1</sup>It should be noted, once more, that the various other (infinite) states corresponding to continuous representations of the  $SL(2, R)$  symmetry that pertain to various tachyon modes do not constitute memory states, because, as mentioned earlier, they are just *excitations* about the ground state.

where  $L_s$  is a characteristic string length (minimum distance).  $\delta L$  is a measurability bound rather than an uncertainty, i.e. it is a quantity that characterizes the matter subsystem in interaction with a measuring apparatus. Thus, the bounds (26) are appropriate in experimental tests of the above theory, where one tries to measure fluctuations in the tips of the MT chains. Eq. (26) is a consequence of a modified dispersion relation for massless photons as a result of the interaction with quantum gravitational degrees of freedom, which in the present model of MT are the  $W_\infty$  global string modes. Such fluctuations are, therefore, length dependent, and constitute an interesting prediction for possible experimental tests.

The above picture of casting the complicated MT dynamics into a simple completely integrable  $1+1$ -dimensional string model in an effectively curved space time (representing the formation of singularities as a result of external stimuli in the brain) opens exciting possibilities for a rigorous study of the formation of such singularities. From a  $1+1$ -dimensional space-time point of view this problem is equivalent to the strongly-coupled problem of black hole formation in two-dimensional string theory. The problem seems complicated, given its strongly-coupled nature. However, recently a new way of treating such kind of problems became possible through the use of generalized duality symmetries, which map the strongly-coupled problem of strings propagating in singular space-time backgrounds into weakly coupled membrane theories, which appear solvable. The membranes used in such an approach are characterized by the incorporation of open strings in their spectrum, with special world-sheet boundary conditions (of Dirichlet type) for a subset of the space-time coordinates of the string. From a critical string point of view this subset characterizes the collective coordinates of the solitonic string background.

It is therefore natural to treat our MT model in this way, with the hope of getting more quantitative information about the formation of the singularities, the recoil of matter due to the fluctuations of space time etc. The two dimensional black-hole problem is a string soliton problem as discussed in ref. [13]. Moreover, in the present MT model, even the flat-space time limit is a kink, characterized by a certain set of collective coordinates. In the past year such complicated string soliton  $\sigma$ -models can be described in an easier way if one employs open strings, i.e.  $\sigma$  models on world-sheets with boundaries, and imposes appropriate boundary conditions for the target-space collective coordinates of the soliton. From a world-sheet point of view a target two-dimensional black hole model represents a defect on the world-sheet [13, 14] and therefore naturally incorporates open strings in its spectrum. The end points of such strings carry the gauge charges of the black-hole  $W_\infty$ -hair carrying information [16]. In this way one has an explicit (in a mathematical sense) way of describing storage of this information on the horizon of the singularity. Details have been given in ref. [16], where we refer the interested reader. From a brain MT viewpoint, the various values of such charges characterize the ground *memory* states, according to our discussion above [24]. The presence of open strings carrying gauge group quantum numbers can also be seen in a totally different way as follows: the



two-dimensional black hole space time has a cigar like structure, but it is symmetric about the spatial origin, i.e it has two asymptotically flat domains. For consistency, one should get rid of one asymptotic domain by some sort of orbifold procedure, i.e. appropriately projecting the target space time of the string onto a single cigar. This induces, according to the analysis of [21], open strings, and leads to an alternative way of representing the two-dimensional black hole as a Dirichlet brane.

Since this meeting, enormous progress has been made in understanding the quantum mechanics of such Dirichlet branes [29]. In our group, we have also made progress towards a formal understanding of several issues arisen here, especially those regarding the connection of microscopic black-hole formation and evaporation with the above-mentioned revolutionary formalism of membrane theories. In particular, a special formalism on the world-sheet of the string has started to be developing with the hope of describing interaction of matter with the membrane/black-hole as accurately as possible. The approach utilizes certain ‘logarithmic operators’ [22, 17], which are special marginal world-sheet operators, capable of describing in a conformal-field-theory setting changes of state of the membrane background. We have actually shown that recoil of the (Dirichlet) string membrane world-volume during scattering of light matter off the membrane induces non-criticality of the pertinent ( $\sigma$ -model) string theory, associated with a change of state of the membrane background. This change of state is described by the logarithmic operators. As discussed in ref. [17], this results in decoherence for the effective subsystem of the light degrees of freedom, due to information carried by the (quantum) recoil degrees of freedom pertaining to fluctuations of the collective coordinates of the soliton [22]. Due to this decoherence, there are modified measurability bounds in lengths (26), implying modified (length-dependent) uncertainty relations in the non-critical string framework. We believe, that this formal approach is useful towards a better understanding of important quantum aspects of black hole physics, which have been argued in this talk and in refs. [28, 24] to play a rôle in brain functioning. The recoil phenomenon, for instance, may be identified with a back-reaction of the spin chain of the MT against an externally induced abrupt change in the respective conformations (*stimulus*). We do hope to come back to such issues in more detail in the near future.

## Acknowledgements

The work of D.V.N. is supported in part by D.O.E. Grant DEFG05-91-GR-40633.

## References

- [1] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Int. J. Mod. Phys. A11 (1997), 607.

- [2] I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, Phys. Lett. B211 (1988), 393; Nucl. Phys. B328 (1989), 117; Phys. Lett. B257 (1991), 278;  
D.V. Nanopoulos, in *Proc. International School of Astroparticle Physics*, HARC (Houston) (World Scientific, Singapore, 1991), p. 183.
- [3] I. Bakas and E. Kiritsis, Int. J. Mod. Phys. A7 (Suppl. 1A) (1992), 55.
- [4] A.O. Caldeira and A.J. Leggett, Ann. Phys. 149 (1983), 374.
- [5] E. Celeghini, M. Rasetti and G. Vitiello, Ann. Phys. (N.Y.) 215 (1992), 156.
- [6] S. Chaudhuri and J. Lykken, Nucl. Phys B396 (1993), 270.
- [7] M.A. Collins, A. Blumen, J.F. Currie, and J. Ross, Phys. Rev. B19 (1978), 3630.
- [8] F. David, Mod. Phys. Lett. A3 (1988), 1651;  
J. Distler and H. Kawai, Nucl. Phys. B321 (1989), 509.
- [9] E. Del Giudice, S. Doglia, M. Milani and G. Vitiello, Nucl. Phys. B251 (FS 13) (1985), 375; *ibid* B275 (FS 17) (1986), 185.
- [10] E. Del Giudice, G. Preparata and G. Vitiello, Phys. Rev. Lett. 61 (1988), 1085.
- [11] P. Dustin, *MicroTubules* (Springer, Berlin 1984);  
Y. Engleborghs, *Nanobiology* 1 (1992), 97.
- [12] J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, Nucl. Phys. B241 (1984), 381.
- [13] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992), 37;  
*lectures presented at the Erice Summer School, 31st Course: From Supersymmetry to the Origin of Space-Time*, Ettore Majorana Centre, Erice, July 4-12 1993 ; hep-th/9403133, Vol. 31 (1994), p.1 (World Sci. ) ;  
For a pedagogical review of this approach see: D.V. Nanopoulos, Riv. Nuov. Cim. Vol. 17, No. 10 (1994), 1.
- [14] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B288 (1992), 23.
- [15] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B276 (1992), 56.
- [16] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, hep-th/9605046; Int. J. Mod. Phys. A, in press.
- [17] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, hep-th/9609238.
- [18] R.P. Feynman and F.L. Vernon Jr., Ann. Phys. (NY) 94 (1963), 118.

- [19] H. Fröhlich, *Bioelectrochemistry*, ed. by F. Guttman and H. Keyzer (Plenum, New York 1986).
- [20] S. Hameroff and R. Penrose, *Orchestrated Reduction of Quantum Coherence in Brain Microtubules: a Model of Consciousness*, in *Towards a science of Consciousness*, The First Tucson Discussions and Debates, eds. S. Hameroff *et al.* (MIT Press, Cambridge MA 1996), p. 507-540.
- [21] P. Horava, Phys. Lett. B289 (1992), 293.
- [22] I. Kogan and N.E. Mavromatos, Phys. Lett. B375 (1996), 111;  
I. Kogan, N.E. Mavromatos and J.F. Wheeler, Phys. Lett. B387 (1996), 483.
- [23] P. Lal, Physics Letters 111A (1985), 389.
- [24] N.E. Mavromatos and D.V. Nanopoulos, ACT-09/95, CTP-TAMU-24/95, ENSLAPP-A-524/95, hep-ph/9505401. Int. J. Mod. Phys. B, in press.
- [25] D. Minic, J. Polchinski and Z. Yang, Nucl. Phys. B369 (1992), 324.
- [26] See for instance: *Gravitation*, C.W. Misner, K.S. Thorne and J.A. Wheeler (W.H. Freeman and Co., San Fransisco (1973)).
- [27] D. Page, *Information Loss in Black Hole and/or Conscious Beings ?*, Alberta preprint, hep-th/9411193, to be published in *Heat Kernel Techniques and Quantum Gravity*, eds. S.A. Fulling (Texas A& M University (1995)).
- [28] R. Penrose, *The Emperor's New Mind* (Oxford Univ. Press 1989); *Shadows of the Mind* (Oxford Univ. Press 1994);  
D.V. Nanopoulos, *Theory of Brain Function, Quantum Mechanincs and Superstrings*, hep-ph/9505374, Proc. "XV Brazilian National Meeting on Particles and Fields", eds. M.S. Alves *et al.*, Brasileam Phys. Society (1995), p. 28.
- [29] For a comprehensiive review on recent developments see: J. Polchinski, *TASI lectures on Dirichlet branes*, hep-th/9611050.
- [30] A.M. Polyakov, Mod. Phys. Lett. A6 (1991), 635.
- [31] L.M. Ricciardi and H. Umezawa, *Kibernetik* 4, (1967) 44.
- [32] M.V. Sataric, J.A. Tuszyński, R.B. Zakula, Phys. Rev. E48 (1993), 589.
- [33] C.I.J. Stuart, Y. Takahashi and H. Umezawa, J. Theor. Biol. 71 (1978) 605; Found. Phys. 9 (1979) 301.
- [34] For a review see H. Umezawa, *Advanced Field Theory : micro, macro and thermal concepts* (American Inst. of Physics, N.Y. 1993).
- [35] G. Vitiello, Int. J. Mod. Phys. B9 (1995), 973.

- [36] E. Witten, Phys. Rev. D44 (1991), 344.
- [37] F. Yu and Y.S. Wu, J. Math. Phys. 34 (1993), 5872.